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On the reverse transition of a turbulent flow under the action of buoyancy forces

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Experiments were conducted in an ascending laminar flow through a vertical pipe under combined free and forced convection at constant heat flux through the wall.

Mean velocity and temperature profiles were measured with a hot-wire probe. This velocity profile which is deformed by the buoyancy forces, enabled us to compute the reduced acceleration parameter. The profiles obtained showed that the value of the parameter at which reverse transition takes place is approximately the same as that found in isothermal boundary-layer flow. By measuring the autocorrelation function of the velocity after the reverse transition it was shown that the flow in the boundary layer becomes laminar as well as fluctuating and that it oscillates with a predominating period.

1. Introduction

In previous experiments by Mreiden (1968) on combined free and forced convection, carried out on the same apparatus, it was found that in a rising flow through a vertical tube, heated by a uniform flux, at values for Gr_0/Re_0 of 13000 and for Re_0 of 10000, the heat exchange is the same as in a laminar flow process, even when the flow is originally turbulent.

$$Gr_0 = gD^4\phi/\lambda_0 T_0 \nu_0^4$$

is the Grashoff number, where D is the pipe diameter, g the gravity acceleration, ϕ the heat flux, λ_0 the heat conductivity at the inlet temperature T_0, and ν_0 the kinematic viscosity at the inlet temperature; Re_0 is the Reynolds number. The present study of the velocity and temperature profiles was undertaken as a continuation of the work described above.

The earliest observations of reverse transition were reported by Wilson & Pope (1954). These concerned the heat exchange in turbine blades. Sibulkin (1962) measured some velocity profiles of gases in pipes, in which the flow passed from a Reynolds number above 3000 to a Reynolds number below 2500 and in which the flow started out as turbulent and then became laminar. By a similar method, Kreith (1965) and Moller (1963) found cases of reverse transition in radially diverging flows between two flat parallel plates.

A different kind of reverse transition was produced by accelerating the flow

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vigorously. Eckert & Rodi (1968) obtained this effect by injecting air through the porous walls of a tube through which air was flowing in a turbulent régime. The acceleration was accompanied by clear evidence of an important fall in pressure.

Launder (1964), Moretti & Kays (1965), Patel & Head (1968), as well as Badri Narayanan & Ramjee (1969) were able to accelerate a flow and make it laminar by using convergents or bodies in a channel.

The reverse transition is usually inferred from the values of global parameters of interchange of momentum or heat at the wall. In this way Moretti & Kays measured the heat exchange in their experimental channel and ascertained that for high values of acceleration the exchange stopped obeying the normal laws of turbulent flow. Further, they correlated this variation of heat exchange with the acceleration parameter $K = (\nu/U^2)\,dU/dx$, in which U is the undisturbed flow velocity and x the longitudinal co-ordinate, and found that the laminarization appeared to arise whenever K exceeded $-3{\cdot}5\times 10^{-6}$. Visual investigations performed by Schraub & Kline (1965) and by Kline and his group have shown that the frequency of the bursts of turbulence leaving the wall is closely related to the parameter K. At around $K = 3{\cdot}5\times 10^{-6}$ the bursts cease.

Patel & Head used an equation, proposed by Townsend (1961), which gives the mean velocities in a flow with a strong pressure gradient. This equation, which is based on the equilibrium between the turbulence generated and dissipated at a given point near the wall, gives explicitly the dependence of the mean velocity on the reduced dimensionless gradient of Reynolds stresses, i.e.

$$\Delta_\tau = (\nu/\rho U^3)\,(\delta\tau/\delta y),$$

where τ are the Reynolds stresses, y the transverse co-ordinate and ρ the specific mass. Patel & Head assumed that laminarization occurs when the foregoing equilibrium is broken at high values of the gradient Δ_τ. According to Patel & Head, the limit of turbulence is determined by a value of the dimensionless gradient of the turbulent stresses.

Bradshaw holds that reverse transition will always take place in a flow which becomes dependent throughout on the viscosity. For this it is necessary that the quotient of the characteristic lengths of the high-energy eddies and the dissipating eddies, which is equal to the turbulent Reynolds number R_τ, nowhere in the flow exceeds the value $30\,k$ where k is von Kármán's constant. Here the turbulent Reynolds number is

$$R_\tau = (\tau/\rho)^{\frac{1}{2}}\,(L/\nu),$$

where L is the dissipation length parameter. Bradshaw (1969) argues that the criterion of Patel & Head can be reduced to his own. With the help of Bradshaw's estimate for the distribution of stresses in terms of the pressure gradient and using an estimated friction factor for an exterior accelerated flow of 0·005, one can show that

$$\{(\tau/\rho)^{\frac{1}{2}}\,(L/\nu)\}_{\max} \leqslant 30k$$

is equivalent to $$K = \nu/U^2\times dU/dX \geqslant -3{\cdot}2\times 10^{-6},$$

which is the criterion proposed by Kline and coworkers, Moretti & Kays and others.

Hall & Jackson (1969) found a phenomenon very similar to the one described in this paper in experiments on heat exchange with vertical ascending flows of CO_2 under conditions close to the critical point and thus in the presence of important buoyancy forces. They explain the reduction of the heat exchange by a reduction of the turbulence owing to the cancellation of the most important term of the turbulent energy generation, i.e. $-\rho\overline{uv} \times dU/dy$, in the zone in which it reaches its highest value. Here u and v are velocity fluctuations. In the Hall & Jackson experiments, as well as in ours, it could be a new distribution of the turbulent stress that cancels the buoyancy forces and that makes

$$\rho\overline{uv} = 0 \quad \text{at} \quad yU_\tau/\nu = 30,$$

where U_τ is the friction velocity.

In the case of strongly accelerated flows it is the pressure gradient which balances the tangential stresses when $yU_\tau/\nu = 30$, i.e. at the point at which $\rho\overline{uv}$ normally reaches its highest value. This scheme allows one to calculate a criterion for the onset of laminarization in vertically ascending flows with significant buoyancy force. Such a scheme is applicable to supercritical flows in which the density varies violently between the wall zone and the core. The criterion under discussion takes the form

$$\beta = Gr/Re^{2\cdot7}\,(\nu_p/\nu_c)^3\,(\rho_c/\rho_p),$$

where

$$Gr = (\rho_c - \rho_p/\rho_c) \times gD^3/\nu_p^2$$

and

$$Re = U_c D/\nu_c.$$

Here subscripts p and c mean physical properties computed at wall and core temperature respectively.

Although in our case, when reverse transition takes place, the very uniform temperature distribution does not allow distinct temperatures to be attributed to the wall region and to the core, the calculation of β on the basis of the temperatures at the wall and at the centre gives values of β of the order of 10^{-4}, which agree with the order of magnitude proposed by Hall & Jackson.

The same scheme applied to accelerated flows justifies a value of the order of -4×10^{-6} for the non-dimensional parameter K.

2. Experimental apparatus

The experimental apparatus (figure 1) consisted of a vertical stainless steel pipe of 8 cm internal diameter and 480 cm in length. The pipe was heated at constant flux by the Joule effect in the wall and insulated by fibre glass shells of 20 cm thickness and by Permaglas flanges at the top and bottom of the pipe. Upstream, air flowed through a 180 cm long unheated pipe that constituted the inlet length. Air entered through a paper filter and passed through a flexible pipe and honeycomb before reaching the inset. Downstream from the heated pipe, air passed through a heat exchanger to be cooled to room temperature before flowing through a sonic nozzle to the suction pump. A set of twenty thermocouples welded to the wall was used to measure the wall temperature. All of these were connected to the same cold junction and joined to the same voltmeter: a Digitec with a high input impedance.

Velocity and temperature were measured with a DISA 55D 53 hot-wire probe connected to a 55D05 DISA anemometer unit. The bridge was operated as a constant-temperature unit for the velocity measurements and as a constant-current unit for the temperature measurements. The probe could be mounted

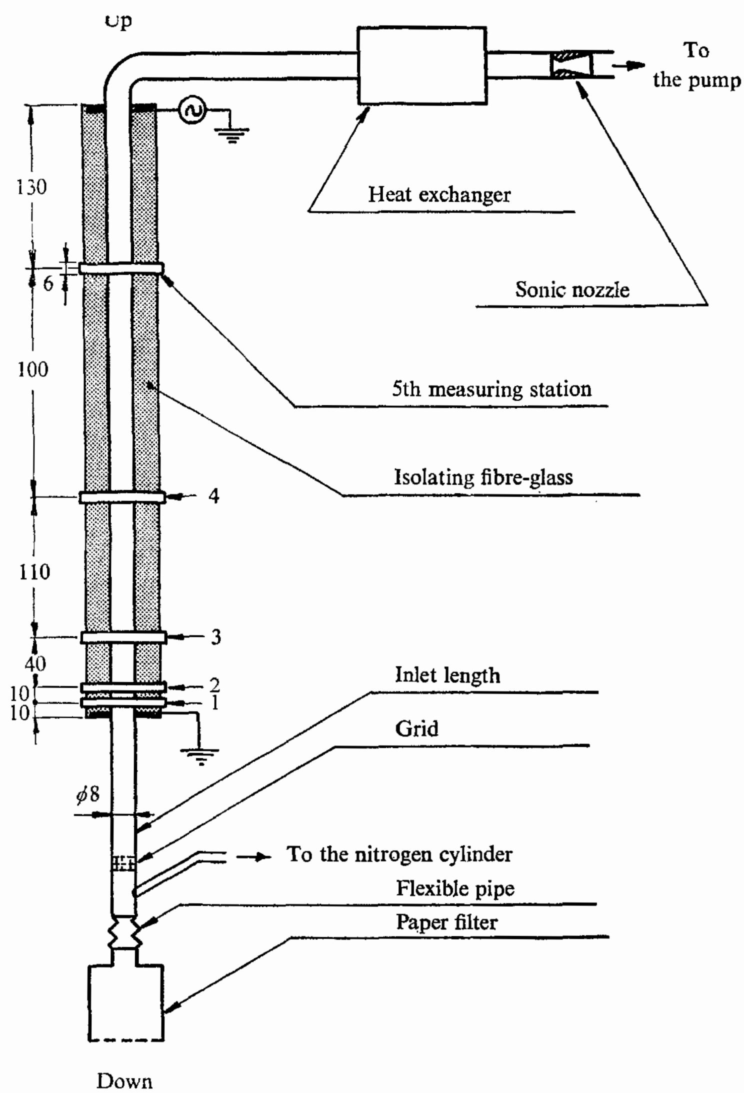

FIGURE 1. Experimental set-up. Scale: 1/40.

in any one of five measuring stations (figure 1). The probes were calibrated for different velocities and at different temperatures. It was especially important to correct for heat losses through the prongs because the form coefficient of the wire used was only 180. The calibration values obtained were very well correlated by the Collis & Williams (1959) formula for the Nusselt number,

$$Nu_f(T_f/T_g)^{-0\cdot17} = 0\cdot24 + 0\cdot56\,(Re_f)^{0\cdot45},$$

in which all physical properties are calculated at film temperature and T_g is the gas temperature. Combining this equation with the well-known correction for heat loss through the prongs, we were able to show that the correct expression for the hot-wire voltage is

$$V = \theta^{\frac{1}{2}} A(1 + [B + CU^{0\cdot45}]^{\frac{1}{2}})$$

for isothermal flow. Here V is the hot-wire voltage; θ the temperature difference between the wire and the gas and U the velocity of the gas.

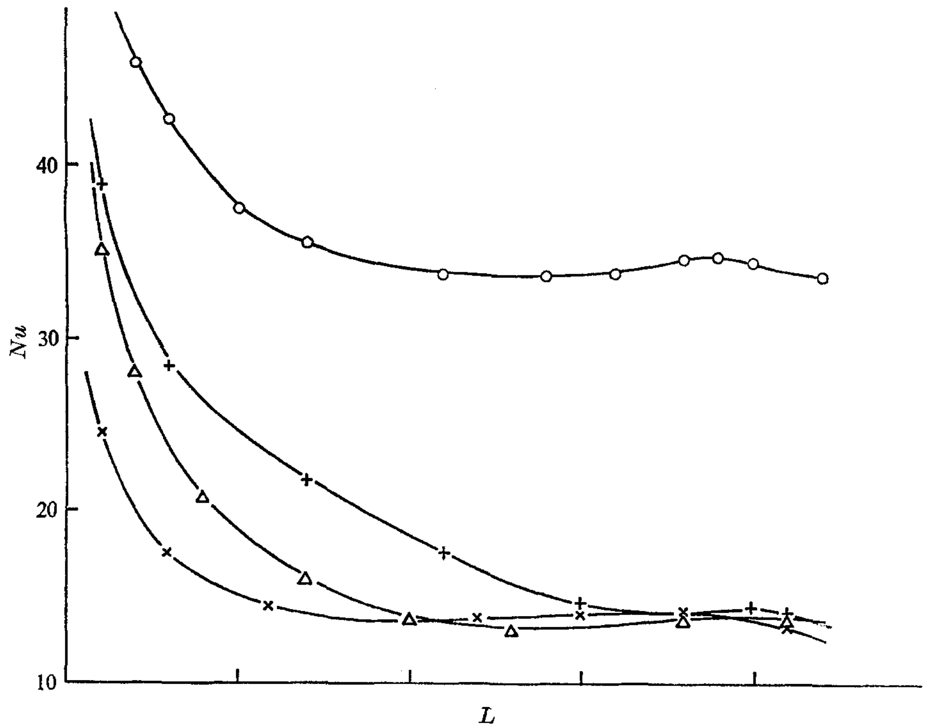

FIGURE 2. Nusselt number variation along the pipe. ○, $Re = 14900$, $Gr/Re = 14550$; +, $Re = 9800$, $Gr/Re = 12400$; △, $Re = 7100$, $Gr/Re = 16490$; ×, $Re = 5000$, $Gr/Re = 18600$.

The expressions for A, B and C, and some details of this calculation, were given by Steiner & Meyer (1970). By comparing the mass flow rate computed by integration of the velocity and temperature profiles to that given by the sonic nozzle, we concluded that our velocity measurements are accurate to within 5 %. The enthalpy flowing through each section, and thus heat flux at the walls, were computed by means of an integration of the temperature and velocity profiles.

3. Results

Figure 2 shows the Nusselt numbers computed from the difference between the wall and mean temperatures as a function of position along the pipe. For Reynolds numbers of 5000, 7100 and 9800, the Nusselt number values are half those given by the usual equations for turbulent flow. It is obvious that reverse transition had occurred in the above three cases but that it did not occur in the fourth case with a Reynolds number of 14700. In the latter case the difference between the experimental Nusselt value and the one given by Hoffmann's formula, quoted by Hinze (1959), is only 2 %.

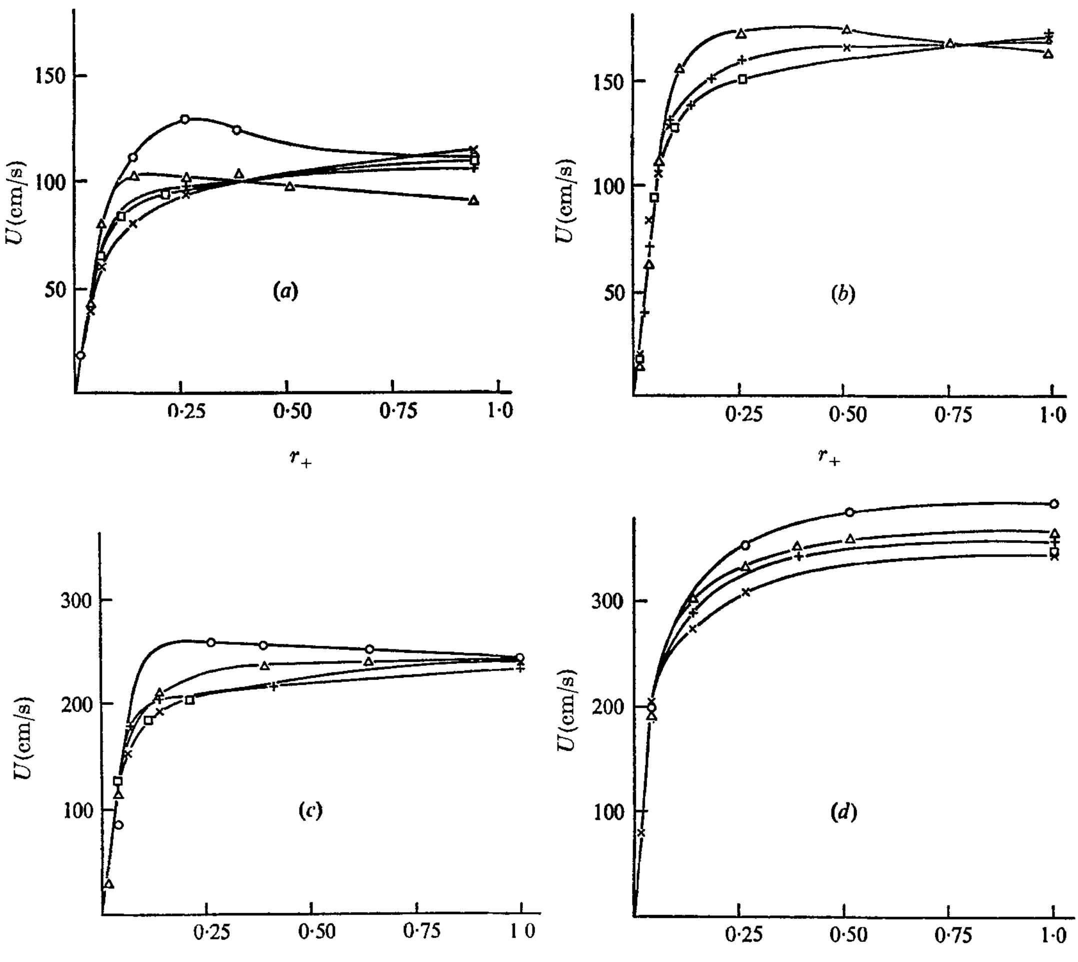

FIGURE 3. Velocity profiles. (*a*) $Re_0 = 5000$, (*b*) $Re_0 = 7100$, (*c*) $Re_0 = 9800$, (*d*) $Re_0 = 14700$. $\times$, station 1; $\square$, station 2; $+$, station 3; $\triangle$, station 4; $\bigcirc$, station 5.

Hallman (1956) showed that under conditions of combined free and forced convection in a fully-developed steady-state ascending laminar flow the solution depends only on G_{r0}/Re_0.

This ratio is more or less the same in the first three examples, where reverse transition occurs, so that the Nusselt number tends to approximately the same value in all three instances. Hallman's solution shows that for high values of Gr_0/Re_0 the maximum velocity does not occur at the centre but near the wall. This tendency is reflected in our velocity profiles shown in figure 3.

In figure 4 we present the temperature profile in the dimensionless form $\theta_+ = \lambda_p (T - T_p)/\phi_0 r_0$, where r_0 is the pipe radius. The differences between the first three profiles and the fourth are very pronounced. For the first three Reynolds numbers the curves tend towards a parabolic shape, characteristic of laminar

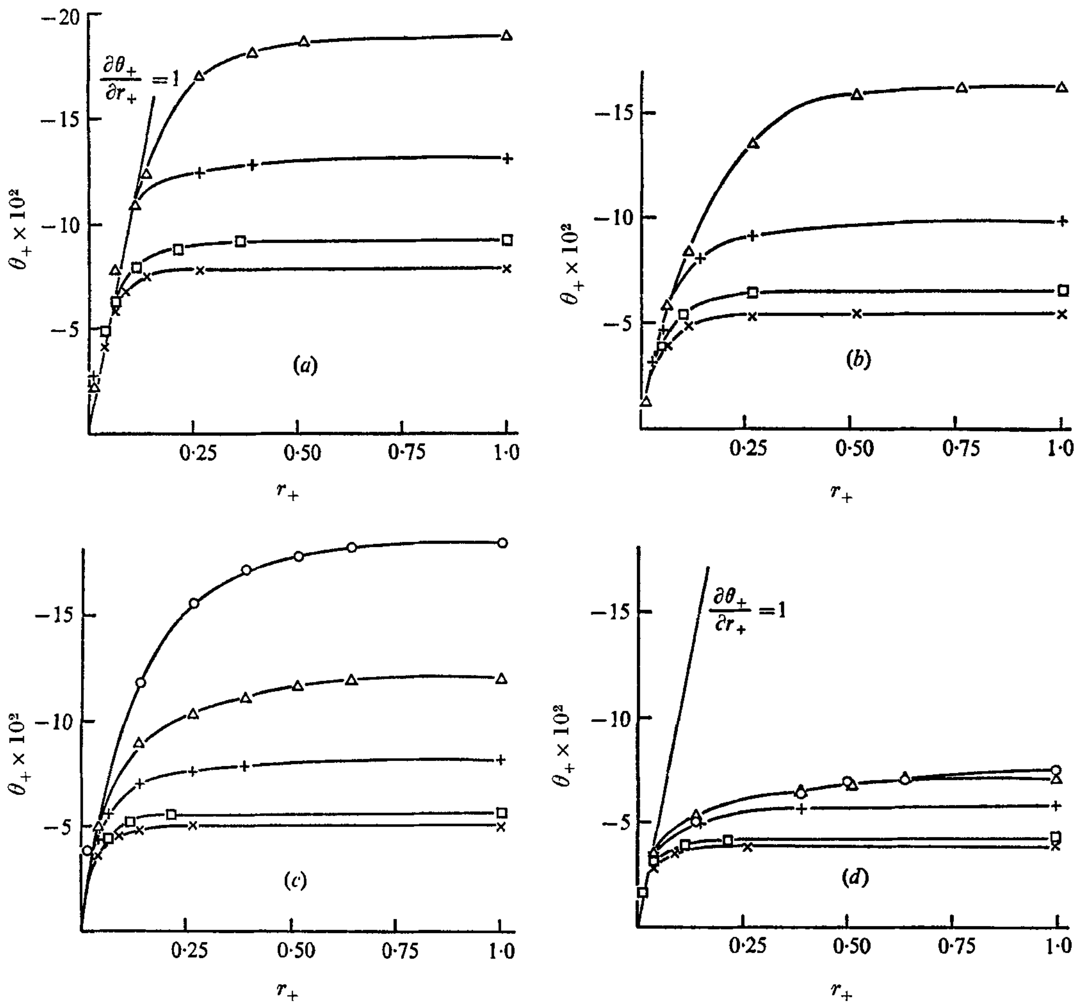

FIGURE 4. Reduced temperatures profiles. $\theta_+ = (T - T\rho)\,\lambda_0/\phi r_0$. (a) $Re_0 = 5000$, (b) $Re_0 = 7100$, (c) $Re_0 = 9800$, (d) $Re_0 = 14700$. See figure 3 for symbols.

flow, while the curves corresponding to $Re_0 = 14700$ display no such tendency and exhibit the flat profiles characteristic of turbulent flow. In the papers by Launder, Moretti & Kays, Patel & Head, Badri Narayanan & Ramjee, referred to in § 1, the laminarization is correlated with the reduced acceleration parameter K belonging to the undisturbed flow. Since ours is an interior flow, there does not exist an undisturbed zone in the strict sense of this term. However, the region of maximum velocity in its relation to the wall region appears to behave like an undisturbed zone in a boundary-layer flow. We computed for each example the dimensionless acceleration parameter K at the radius where it reaches its greatest value on the basis of the longitudinal velocity gradient calculated between the last two stations (table 1).

It is remarkable that the K-parameter test used by Moretti & Kays, as well as Badri Narayanan & Ramjee, for their experiments, seems to be a good criterion for reverse transition even for our very different experimental arrangement. The error in K stems from the 5 % error in the velocity measurements. Although there

Re_0	r_+	dU/dx	U	K
5000	0·30	28/180	100	$-3{\cdot}1 \times 10^{-6}$
7100	0·30	28/110	160	$-1{\cdot}4 \times 10^{-6}$
9800	0·30	48/180	230	$-1{\cdot}1 \times 10^{-6}$
14700	1·00	42/180	350	$-0{\cdot}3 \times 10^{-6}$

TABLE 1

is a zone where the velocity profile shows an inflexion point, the existence of the latter does not seem to have any influence on the reverse transition. It is important to notice that previous experiments carried out with our apparatus by Mreiden showed that the longitudinal pressure gradient changes sign along the pipe.

4. Correlation measurements

The effect of temperature fluctuation on the bridge voltage during the velocity measurement was more important than that of the velocity fluctuation. For this reason our apparatus was modified so that nitrogen could be blown from a cylinder at a steady mass-flow rate throughout the velocity measurements. It was necessary to insert a honeycomb above the entrance opening for the nitrogen in order to prevent the formation of a swirling flow that occurred in the absence of the honeycomb. This swirling flow interfered with the laminarization as could be observed from a drop in the wall temperatures. We heated our oxydizable tungsten wire to 600 °C in order to render it unresponsive to temperature fluctuations. Several runs with a Reynolds number of 7230 were made and we recorded the bridge signal for 4 different locations of the probe in the fifth station. Afterwards the data were analyzed by a DIDAC 800 correlator (a small INTERTECHNIQUE computer), which produced the estimate of the longitudinal velocity autocorrelation function

$$\hat{R}(\tau) = (1/T)\int_0^T U(t) \times U(\tau+t)\,dt.$$

The results were displayed as a set of 390 discrete values, 0·0059 sec apart. The integration time T was 300 sec. Figure 5 (plate 1) shows the graphs of $\hat{R}(\tau)$ at stations 0·57 mm, 2·57 mm, 4·57 mm and 29·57 mm from the wall. The graphs are comparable as the ordinate $\hat{R}(0)$ is the same for all four. The graph of $\hat{R}(\tau)$ at a distance of 0·57 mm from the wall does not resemble the usual graphs of a turbulent flow autocorrelation function. Instead, there appears a sinusoidal curve super-imposed on a normal turbulent autocorrelation function. We computed the estimated variance at the relative maxima and minima of the curve. This variance showed that $\hat{R}(\tau)$ actually possessed these maxima and minima. The period of the sine curve is about 8 Hz, as can also be seen in the graphs taken at 2·57 mm and 4·57 mm from the wall. A 21 Hz period, instead of an 8 Hz period, was recorded near the axis of the pipe. This corresponds to the stationary sonic wave formed in the pipe.

The observed fluctuations in the laminar flow near the wall may be attributed to more than one cause. First, the turbulence of the fluid changes its character during laminarization, as is clear from our graphs of $\hat{R}(\tau)$, but may not be dissipated immediately. In addition, the fluctuations may result partly from the disturbance of the boundary of the laminarized zone due to the turbulent core.

5. Conclusions

The results presented here may serve to establish the following points concerning the inadequately understood mechanism of reverse transition: (*a*) The reverse transition would appear to be primarily due to the acceleration away from the wall region. The part played in this phenomenon by other factors, such as Δ_τ is not entirely clear. (*b*) In our experiments the acceleration rate K was fixed by the relation between Gr_0/Re_0 and Re_0. High values of this parameter led to reverse transition. (*c*) A disturbance of the formation of the accelerated zone by a bulk swirling of the flow was found to prevent reverse transition. (*d*) Reverse transition was invariably encountered only near the wall. (*e*) Our autocorrelation measurements suggest that the fluctuations occurring after laminarization are not turbulent in character. We believe that the change from the turbulent to the laminar régime is not a gradual one.

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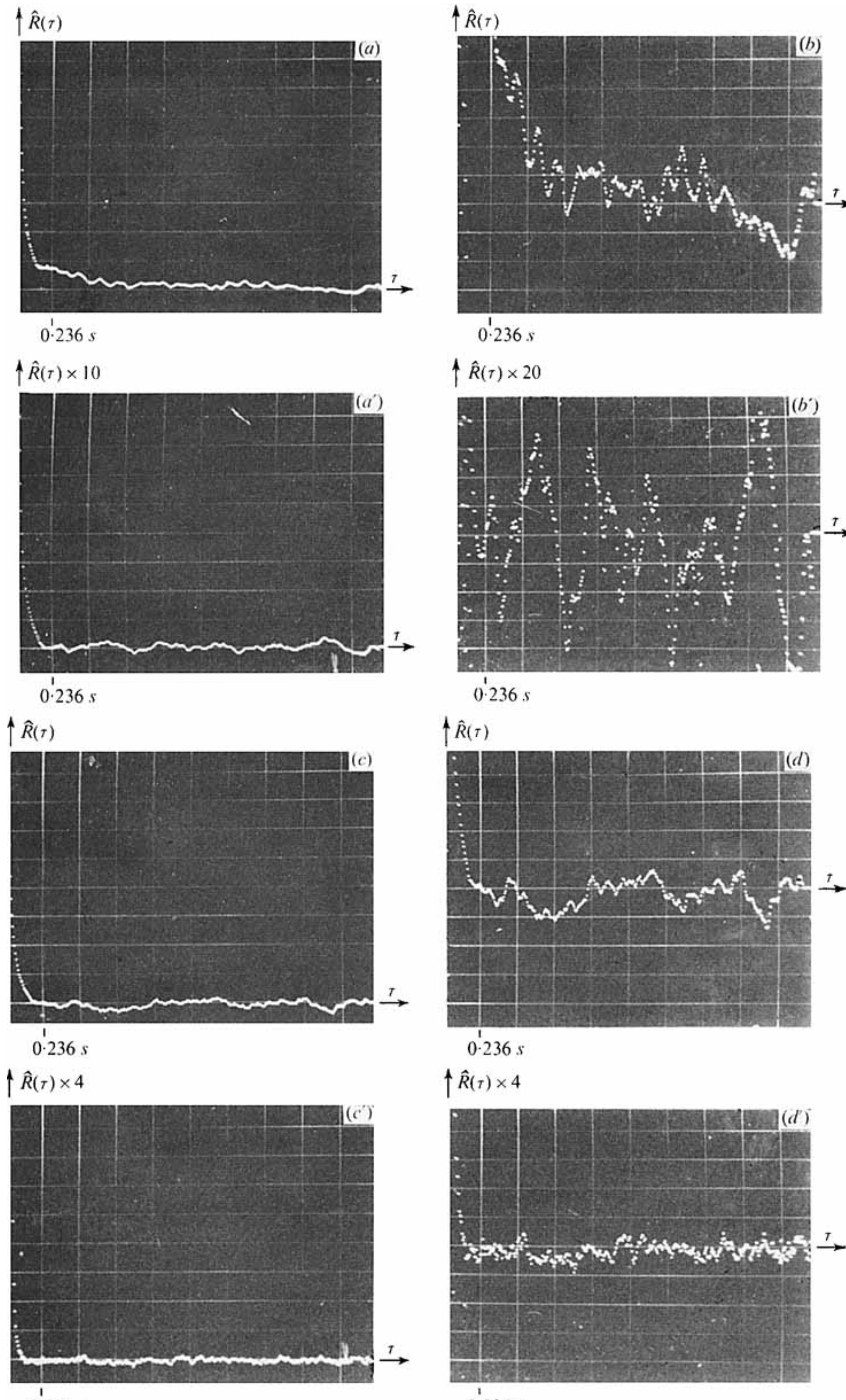

FIGURE 5. Autocorrelation function estimates. (a) $\hat{R}(\tau)$ at 0·57 mm. (a') $10\hat{R}(\tau)$ at 0·57 mm. (b) $\hat{R}(\tau)$ at 2·57 mm. (b') $20\hat{R}(\tau)$ at 2·57 mm. (c) $\hat{R}(\tau)$ at 4·57 mm. (c') $4\hat{R}(\tau)$ at 4·57 mm. (d) $\hat{R}(\tau)$ at 24·57 mm. (d') $4\hat{R}(\tau)$ at 24·57 mm.